1 Dynamical CPA $\Longleftrightarrow$ DMFT

1.1 Hubbard Alloy Analogy

Naturally generalizes to to a dynamical, eg. quantum alloy, analogy

The Hubbard Model:

$$\widehat{H} = \sum_{i,j;\sigma} [(\epsilon - \epsilon_0)\delta_{i,j} - t_{i,j}] c^{\dagger}_{i,\sigma} c_{j,\sigma} + \frac{1}{2} U \sum_{i,\sigma} c^{\dagger}_{i,\sigma} c_{i,\sigma} c^{\dagger}_{i,-\sigma} c_{i,-\sigma}$$

$$\sum_j [(\epsilon - \epsilon_{i,\sigma}^{HF})\delta_{i,l} + t_{i,l}] G_{\sigma,\sigma}^{HF}(l, j; \epsilon) = 0$$

$$\epsilon_{i,\sigma}^{HF} = \epsilon_0 + U\overline{n}_{i,-\sigma}$$

$$n_{i,\sigma} = \xi_i = \begin{matrix} 0 \\ 1 \end{matrix}$$

1.2 Hubbard III Approximation=CPA:

$$G_{\sigma,\sigma}(l, j; \epsilon) = \left\langle G_{\sigma,\sigma}^{HF}(l, j; \epsilon; \{n_{i,-\sigma}\}) \right\rangle_{CPA}$$

$$\frac{1}{2}(1+m)G^{\Uparrow}_{\sigma,\sigma;imp}(i,i;\epsilon)+\frac{1}{2}(1-m)G^{\Downarrow}_{\sigma,\sigma;imp}(i,i;\epsilon)=G^{c}_{\sigma,\sigma}(i,i;\epsilon)$$

$$m=\frac{1}{N}\sum_{i}^{N}(\overline{n}_{i,\Uparrow}-\overline{n}_{i,\Downarrow})$$

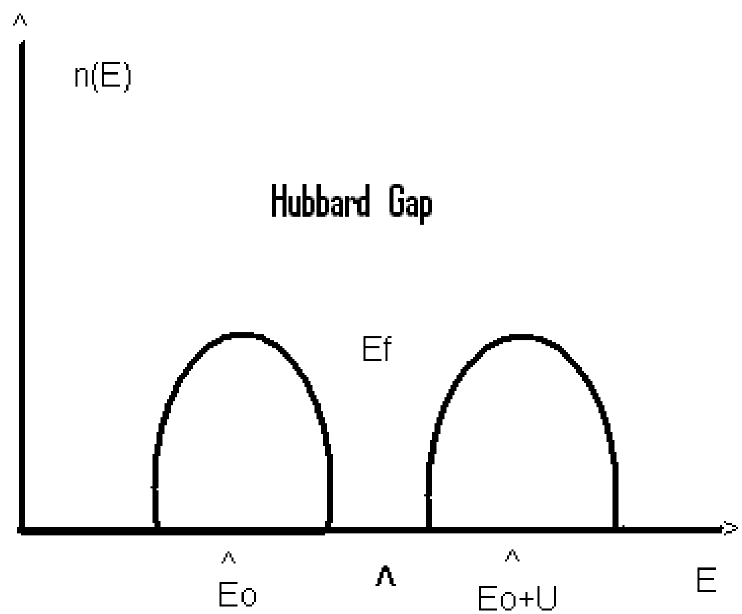
$$\overline{n}_{i,\sigma}(\epsilon)=\frac{1}{\pi}\,\mathsf{Im}<G_{\sigma,\sigma}(i,i;\epsilon)>$$

It may turn ou to be the case that

$\overline{n}_{i,\Uparrow}(\epsilon)-\overline{n}_{i,\Downarrow}(\epsilon)\neq 0$ this means *band spliting.*

even for m=o , that is to say T$\geq T_{c.}$.and there exsist a local moment

$$\mu = \mu_B \int d\epsilon (\overline{n}_{i\Uparrow}(\epsilon) - \overline{n}_{i\Downarrow}(\epsilon)) \geq 0$$

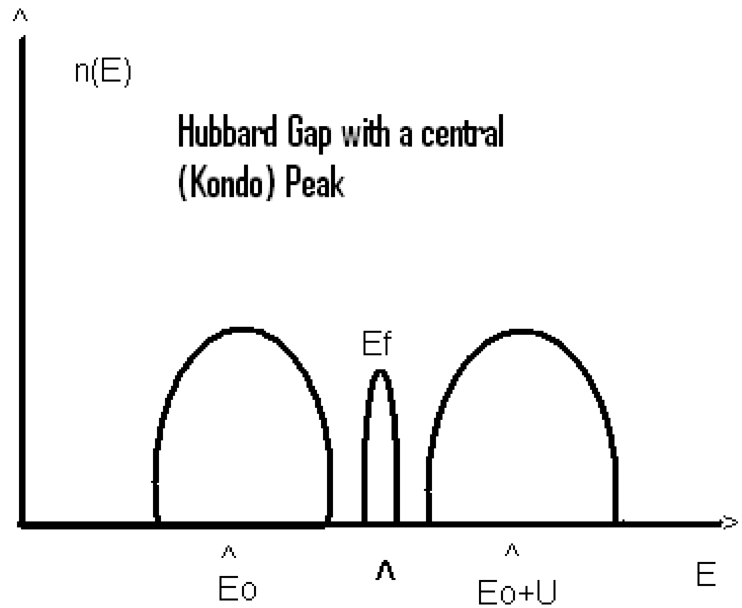

n(E)
Hubbard Gap
Ef
Eo
Eo+U
E

1.3 Dynamical CPA=Dynamical Mean Field (DMFT)

(Kakehashi ,Phys.Rev.B 45 7196,**1992** A.Georges and G.Kotliar Phys.Rev.B 45, 6479, (**1992**))

$$\epsilon_{i,\sigma} = \epsilon_0 + U n_{i,-\sigma} \Rrightarrow \epsilon_0 + \mathbf{v}_{i.-\sigma}(\tau)$$

$$\underline{G}_{i,i}(\tau,\tau') = \frac{1}{Z} \prod_{i,\sigma} \int D\mathbf{v}_{i.-\sigma}(\tau) \exp\{-\frac{1}{U} \sum_{i,\sigma} \int d\tau \mathbf{v}^2_{i.-\sigma}(\tau)\} \underline{G}_{i,i}(\tau,\tau';[\mathbf{v}_{i.-\sigma}(\tau)])$$

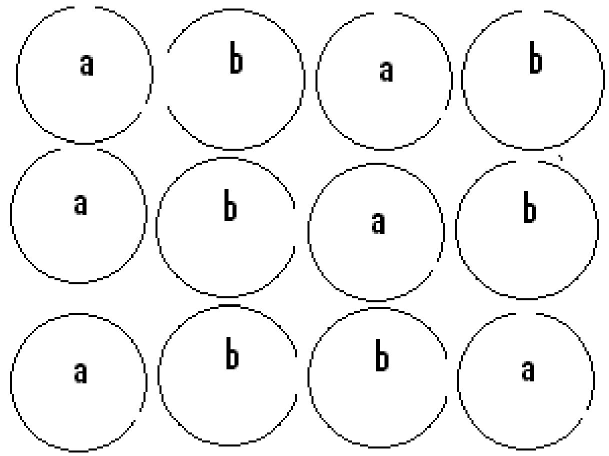

n(E)
Hubbard Gap with a central
(Kondo) Peak
Ef
Eo
Eo+U
E

2 First principles Alloy analogy

2.1 FLUCTUATIONS

in Density Functional theories can be described as fallows:

Using Constrained Density Functional Theory force the potential wells at each site to be either $\mathsf{v}_a(\overrightarrow{r} - \overrightarrow{R}_i)$ or $\mathsf{v}_b(\overrightarrow{r} - \overrightarrow{R}_i)$:



Alloy Analogy Potential Wells

THEN summm over the sconstraints.

2.2 Summing over CONSTRAINTS

In KKR

$$\mathsf{v}_a \Rrightarrow \mathsf{f}_l^a(\epsilon) = \frac{1}{2i}\left(e^{i2\delta_l^a} - 1\right)$$

$$\mathsf{v}_b \Rrightarrow \mathsf{f}_l^b(\epsilon) = \frac{1}{2i}\left(e^{i2\delta_l^b} - 1\right)$$

and the summ over STATIC CONSTRAINTS can be done by *KKR-CPA* .

2.3 Missing from this discussion:

Short range Order

and

Dynamics

3 Dynamical (quantum) Alloy Analogy

3.1 Dynamical versus quantum

description of fluctuations:

$$Dynamics \Rrightarrow \int Dv(\overrightarrow{r},\tau)e^{-S_{eff}[\mathsf{v}(\overrightarrow{r},\tau)]}$$

$$Quantum\ \text{alloy} \Rrightarrow Tr\{e^{-\beta H_{eff}(\widehat{S}_x,\widehat{S}_y\widehat{S}_z)}\}$$

3.2 Internal Two Level Systemes

External TLS in metals, like an atom tunneling between two equivalent positions in a metallic glass,have been studied for a long time.As was first pinted out by J.Kondo their physics has much in common with magnetic impurities in transition metals ,such as Fe in Au. A good review of the theory is by Kondo himself in "Fermi surface Effects" by J.Kondo and Yoshimori (Eds.) (Springr Series in Solid-State Sciences 77 ,Springer -Verlag 1987)Here I propose to postulate that the local potential v($\overrightarrow{r} - \overrightarrow{R}_i; \eta_i$) seen by an electron depends not only on $\overrightarrow{r}$ but also a new ,local, INTERNAL variable η which describes the state of an 'atom' on the site concerned. Such state of the 'site' is described by

$$H_{TLS}\Phi_a(\eta) = \epsilon_a \Phi_a(\eta)$$

$$\Phi_a(\eta) =< \eta \mid \Phi_a >$$

$$\widehat{H}_{TLS} \mid \Phi_a >= \epsilon_a \mid \Phi_a >$$
$$\widehat{H}_{TLS} \mid \Phi_b >= \epsilon_b \mid \Phi_b >$$

$$\mid \Phi_a >= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mid \Phi_b >= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\widehat{S}_z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$\widehat{H}_{TLS} = \epsilon_a \frac{1}{2}(1 + \widehat{S}_z) + \epsilon_b \frac{1}{2}(1 - \widehat{S}_z)$$

3.3 Classical TLS

$\widehat{S}_{z,i}$ commutes with $\widehat{H}_{TLS,i}$ for all sites labeled by i. Thus the collective state of the internal TLS s (one TLS for each site) can be defined by assigning an eigen value of $\widehat{S}_{z,i}$, namely $\mathrm{M}_i = \pm 1$, to each site.If

$$\widehat{H}_{electrons} = \widehat{H}_0 + \sum_i \hat{\mathsf{v}}_{a,i}\frac{1}{2}(1+\widehat{S}_{z,i}) + \hat{\mathsf{v}}_{b,i}\frac{1}{2}(1-\widehat{S}_{z,i})$$

an energy eigen state of the combined , electrons+ TLSs sytem, $\mid n >$, is defined by

$$\left(\widehat{H}_{electrons} + \sum_i \widehat{H}_{TLS,i}\right) \mid n >= E_n(\{M_i\} \mid n >$$

and the partition function Z is

$$\begin{aligned} Z &= tr\{e^{-\beta(H_{electrons}+\sum_i \widehat{H}_{TLS,i})}\} \\ &= \sum_{\{M_i\}} \sum_n e^{-E_n(\{M_i\})} = Z_{TLS} < Z(\{M_i\} > \end{aligned}$$

where

$$Z_{TLS} = \sum_{\{M_i\}} e^{-\beta \sum_i \left(\epsilon_a \frac{1}{2}(1+M_i)+\epsilon_b \frac{1}{2}(1-M_i)\right)}$$

and $<$O($\{$M$_i\}$) $>$ is the average of O($\{$M$_i\}$) over the configurations $\{M_i\}$ with the weight

$$P_{TLS}(\{M_i\}) = \frac{1}{Z_{TLS}} e^{-\beta \sum_i \left(\epsilon_a \frac{1}{2}(1+M_i)+\epsilon_b \frac{1}{2}(1-M_i)\right)}$$

Evidently this is the usual, classical, alloy picture.When $\epsilon_a = \epsilon_b$ all configurations have equal wight. Interstingly,it is also the beginning of the 'agmented space formalism' of Mookerjee.

3.4 Quantum TLS

To 'quantize' the above (Ising) spin system we introduce,as well as $\widehat{S}_z$, the other components, $\widehat{S}_x\ and\ \widehat{S}_y$, of the spin (1/2) operator $\overrightarrow{S}$.

$$\widehat{S}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \widehat{S}_y = \begin{pmatrix} 0 & -i \\ i & o \end{pmatrix}$$

Equivalent but more usefull in the present context are $\widehat{S}_z$, S^+, S^- where

$$S^{\pm} = \widehat{S}_x \pm i\widehat{S}_y$$

$$S^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, S^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Evidently,at each site i

$$S_i^{+} \mid a, i >= 0, S_i^{-} \mid a, i >=\mid b, i >$$

$$S_i^{+} \mid b, i >=\mid a, i >, S_i^{-} \mid b, i >= 0$$

The collective state of the TLSs is

$$\mid \{M_i\} >=\mid M_1, M_2, ...M_i... >= \Pi_i \mid M_i >$$

Thus the full Hamiltonian is

$$\widehat{H} = \widehat{H}_0 + \widehat{H}_{TLS} + \widehat{V}_a + \widehat{V}_b + \widehat{V}_+ + \widehat{V}_-$$

$$\widehat{V} = \sum_i \widehat{\mathsf{v}}_i$$

$$\widehat{\mathsf{v}}_{a,i} = \int d^3\Psi_s^\dagger(\widehat{r})\frac{1}{2}\left(\widehat{1} + \widehat{S}_{z,i}\right)\mathsf{v}_{s,s'}^{a,a}(\widehat{r} - \widehat{R}_i)\Psi_{s'}(\widehat{r})$$

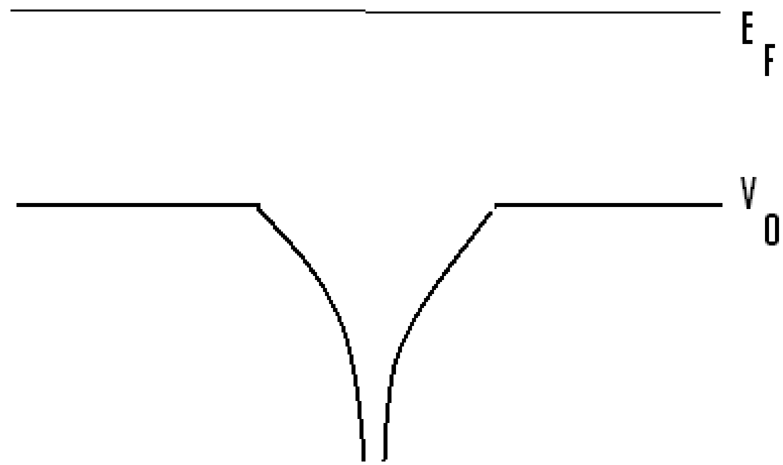
$$\widehat{\mathsf{v}}_{b,i} = \int d^3\Psi_s^\dagger(\widehat{r})\frac{1}{2}\left(\widehat{1} - \widehat{S}_{z,i}\right)\mathsf{v}_{s,s'}^{b,b}(\widehat{r} - \widehat{R}_i)\Psi_{s'}(\widehat{r})$$

$$\widehat{\mathsf{v}}_{+,i} == \int d^3 \Psi_s^\dagger(\widehat{r}) \frac{1}{2} S_+ \mathsf{v}_{s,s'}^{a,b}(\widehat{r} - \widehat{R}_i) \Psi_{s'}(\widehat{r})$$

$$\widehat{\mathsf{v}}_{-,i} == \int d^3 \Psi_s^\dagger(\widehat{r}) \frac{1}{2} S_- \mathsf{v}_{s,s'}^{b,a}(\widehat{r} - \widehat{R}_i) \Psi_{s'}(\widehat{r})$$

3.5 Many-Particle Single Scatterer Theory

A single (muffin-tin type) potential well scatters a degenerate system of non-interacting electrons:



Scattering of a degenerate electron sytem from a potential well

The ususal one-particle t-matrix t$_{\vec{k}\,', \vec{k}}(\epsilon)$ generalizes to

$$T_{\overrightarrow{k}'s';\overrightarrow{k},s}(E) = < \Psi_{\overrightarrow{k}',s'} \mid \widehat{T}(E) \mid \Psi_{\overrightarrow{k},s} >$$

where

$$\widehat{T}(E) = \widehat{V} + \widehat{V}\widehat{G}_0(E)\widehat{T}(E)$$

$$\widehat{G}_0(E) = \frac{1}{E - \widehat{H}_0}$$

$$\widehat{H}_0 = \sum_{\widehat{k},s} \epsilon_{\overrightarrow{k},s} c^{\dagger}_{\widehat{k},s} c_{\overrightarrow{k},s}$$

$$| \Psi_{\vec{k},s} >= c^{\dagger}_{\vec{k},s} \mid \Psi_0 >$$

$$\begin{aligned} \widehat{V} &= \sum_{\vec{k}',s',\vec{k},s} \mathsf{v}^{I}_{\vec{k}'s';\vec{k},s} \widehat{S}_I c^{\dagger}_{\vec{k}',s'} c_{\vec{k},s} \\ I &= z, \pm \end{aligned}$$

Clearly, for $\mathsf{v}^{z}_{\vec{k}'s';\vec{k},s} \neq 0$ but $\mathsf{v}^{\pm}_{\vec{k}'s';\vec{k},s} = 0$ we recover the usual KKR-CPA.

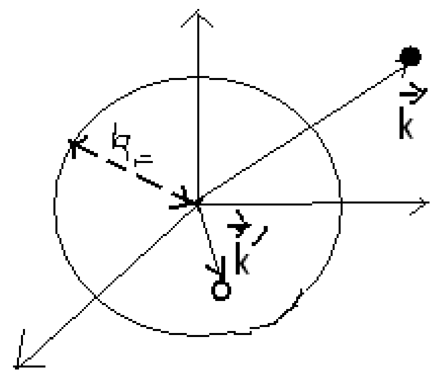
$$\begin{aligned} \mathsf{v}^{\pm}_{\vec{k}'s';\vec{k},s} &\neq 0 \\ Quantum\ Alloy &\Longrightarrow < M \pm 1 \mid \widehat{S}_{\pm} \mid M > \neq 0 \\ \epsilon_{M\pm 1} - \epsilon_M &\neq 0 \\ &\Longrightarrow Dynamics \end{aligned}$$

Perturbation theory::

$$\widehat{T}(E) = \widehat{V} + \widehat{V}\frac{1}{E-\widehat{H}_0}\widehat{V} + \widehat{V}\frac{1}{E-\widehat{H}_0}\widehat{V}\frac{1}{E-\widehat{H}_0}\widehat{V}.....$$

3.5.1 Particles and holes

$$\begin{aligned}
C^{\dagger}_{\overrightarrow{k}} &= \alpha^{\dagger}_{\overrightarrow{k}} \quad \text{for } \left|\overrightarrow{k}\right| > \text{ k}_F \\
C^{\dagger}_{\overrightarrow{k}} &= \beta_{-\overrightarrow{k}} \quad \text{for } \left|\overrightarrow{k}\right| < \text{ k}_F \\
C_{\overrightarrow{k}} &= \alpha_{\overrightarrow{k}} \quad \text{for } \left|\overrightarrow{k}\right| > \text{ k}_F \\
C_{\overrightarrow{k}} &= \beta^{\dagger}_{-\overrightarrow{k}} \quad \text{for } \left|\overrightarrow{k}\right| < \text{ k}_F
\end{aligned}$$



$$E_{\vec{k}} > E_F$$

$$E_{\vec{k}'} < E_F$$

Particles Holes

$$\widehat{H}_0 - \mu\widehat{N} = \sum_{\overrightarrow{k}} \epsilon_{\overrightarrow{k}} C^{\dagger}_{\overrightarrow{k}} C_{\overrightarrow{k}} - \mu \sum_{\overrightarrow{k}} C^{\dagger}_{\overrightarrow{k}} C_{\overrightarrow{k}} = \sum_{\overrightarrow{k}.>k_F} \epsilon_{\overrightarrow{k}} \alpha^{\dagger}_{\overrightarrow{k}} \alpha_{\overrightarrow{k}}$$

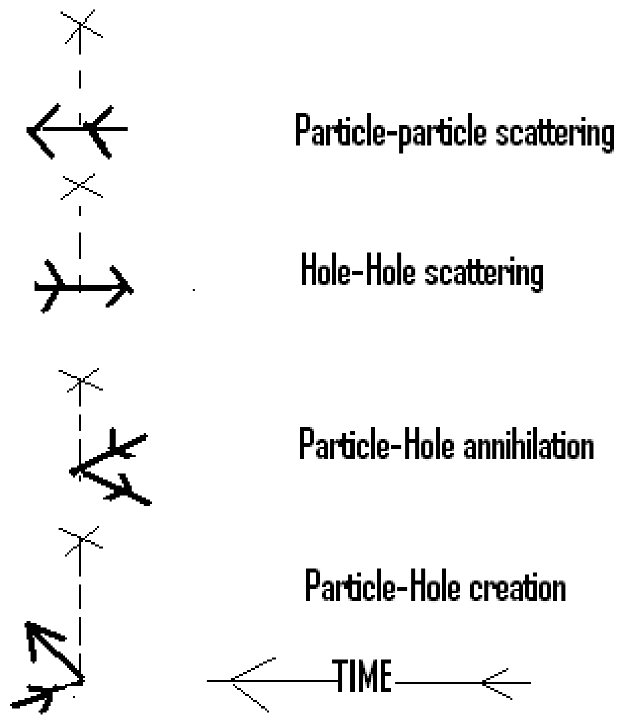
$$+ \sum_{\overrightarrow{k}.<k_F} \epsilon_{\overrightarrow{k}} \beta_{\overrightarrow{k}} \beta^{\dagger}_{\overrightarrow{k}} \quad -\mu \sum_{\overrightarrow{k}.>k_F} \alpha^{\dagger}_{\overrightarrow{k}} \alpha_{\overrightarrow{k}} - \mu \sum_{\overrightarrow{k}.<k_F} \epsilon_{\overrightarrow{k}} \beta_{\overrightarrow{k}} \beta^{\dagger}_{\overrightarrow{k}}$$

$$= \sum_{\overrightarrow{k}<k_F} (\epsilon_{\overrightarrow{k}} - \mu) + \sum_{\overrightarrow{k}.>k_F} (\epsilon_{\overrightarrow{k}} - \mu) \alpha^{\dagger}_{\overrightarrow{k}} \alpha_{\overrightarrow{k}} +$$

$$\sum_{\overrightarrow{k}.<k_F} (\mu - \epsilon_{\overrightarrow{k}}) \beta^{\dagger}_{\overrightarrow{k}} \beta_{\overrightarrow{k}}$$

$$\widehat{V} = \sum (\mathsf{v}_{\overrightarrow{k}', \overrightarrow{k}} \alpha^{\dagger}_{\overrightarrow{k}} \alpha_{\overrightarrow{k}} + \mathsf{v}_{\overrightarrow{k}', \overrightarrow{k}} \alpha^{\dagger}_{\overrightarrow{k}'} \beta \dagger_{-\overrightarrow{k}} + \beta_{-\overrightarrow{k}'} \alpha_{\overrightarrow{k}} + \mathsf{v}_{\overrightarrow{k}', \overrightarrow{k}} \beta_{-\overrightarrow{k}'} \beta^{\dagger}_{-\overrightarrow{k}}$$

Diagrammatically

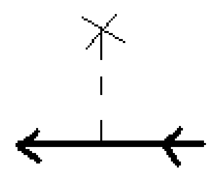


3.5.2 Scattering from a static potential $\mathsf{v}_{\vec{k}\,',\vec{k}}$:

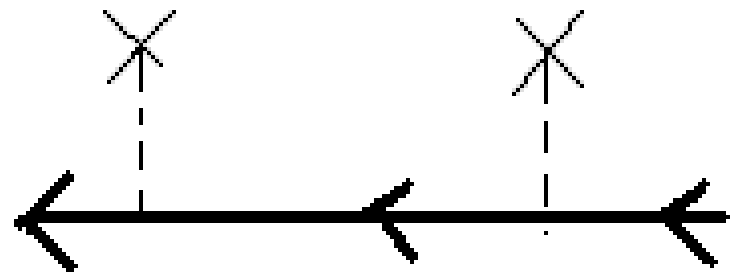
For spinless 'particle-particle'scattering

$$| \Psi_{\vec{k}} >= \alpha^{\dagger}_{\vec{k}} \, | \Psi_0 >$$

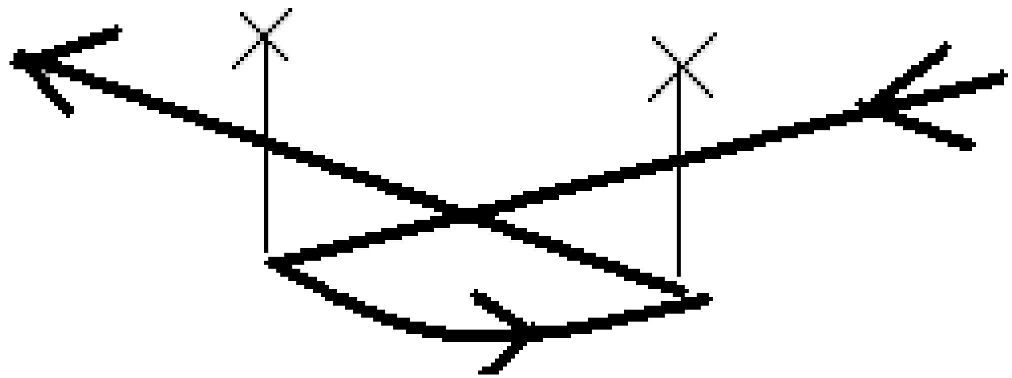
$$T^{(1)}_{\vec{k}\,',\vec{k}}(\epsilon_{\vec{k}}) =$$



$$= \mathsf{v}_{\vec{k}\,',\vec{k}}$$

$$T^{2a}_{\vec{k}',\vec{k}}(\epsilon_{\vec{k}}) =$$ 

$$= \sum_{\vec{k}''} \mathsf{v}_{\vec{k}',\vec{k}''} \frac{1 - f_{\vec{k}''}}{\epsilon_{\vec{k}} - \epsilon_{\vec{k}''}} \mathsf{v}_{\vec{k}'',\vec{k}}$$



$$T^{2b)}_{\overrightarrow{k}',\overrightarrow{k}}(\epsilon_{\overrightarrow{k}}) \;=\; -\sum_{\overrightarrow{k}''} \mathsf{v}_{\overrightarrow{k}'',\overrightarrow{k}} \frac{f_{\overrightarrow{k}''}}{-\epsilon_{\overrightarrow{k}'} + \epsilon_{\overrightarrow{k}''}} \mathsf{v}_{\overrightarrow{k}',\overrightarrow{k}''}$$

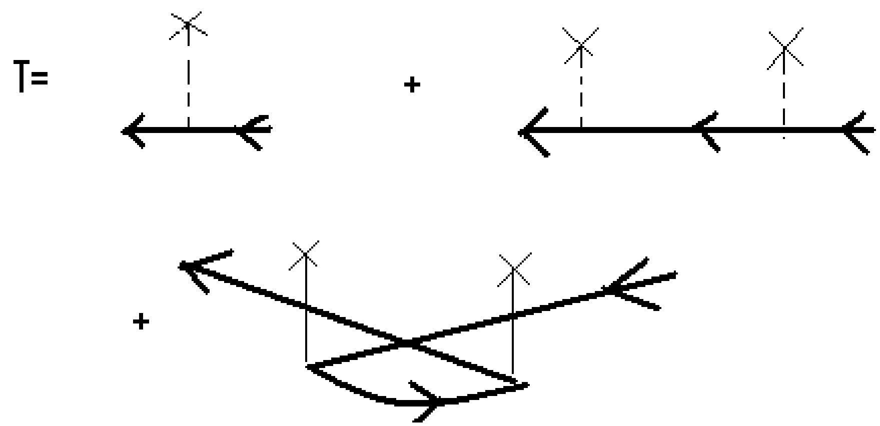
$$T^{2a}_{\overrightarrow{k}',\overrightarrow{k}}(\epsilon_{\overrightarrow{k}}) + T^{2b)}_{\overrightarrow{k}',\overrightarrow{k}}(\epsilon_{\overrightarrow{k}}) =$$

$$\mathsf{v}_{\vec{k}',\vec{k}} + \int d^3k'' \mathsf{v}_{\vec{k}',\vec{k}''} \frac{1}{\epsilon_{\vec{k}} - \epsilon_{\vec{k}''}} \mathsf{v}_{\vec{k}'',\vec{k}}$$

$$+[-\int d^3k'' \left(\mathsf{v}_{\vec{k}',\vec{k}''} \mathsf{v}_{\vec{k}'',\vec{k}} - \mathsf{v}_{\vec{k}'',\vec{k}} \mathsf{v}_{\vec{k}',\vec{k}''} \right) f_{\vec{k}''} \Longrightarrow 0]$$

This happens to all orders and hence $\mathsf{T}_{\vec{k}',\vec{k}}(\epsilon_{\vec{k}})$ is given by the convetional Born series for one particle scattering.If the target can change ist state and therefore, in general,its energy the scattering electon will be knocked off the energy shell .Namely, electron hole pairs will be created resulting in the famous Kondo divergences.

$$T = V + VGT$$



Feynmann Diagram for Kondo Scattering

3.5.3 The break down of the Born peturbation series

$$T^{(1)}_{\vec{k'}M',\vec{k}M}(E) = \sum_I V^I_{\vec{k'},\vec{k}} S^I_{M',M}$$

$$\begin{aligned}
\widehat{T}^{(2)}_{\vec{k}'M';\vec{k},M}(\epsilon_{\vec{k}}) \;&=\; \sum_{I,J}\sum_{M''}\sum_{\vec{k''}} V^I_{\vec{k'}\vec{k''}} S^I_{M',M''} V^J_{\vec{k}'',\vec{k}} S^J_{M'',M} \frac{1-f_{\vec{k}''}}{\epsilon_{\vec{k}'} - \epsilon_{\vec{k}''}} \\
&\quad - V^I_{\vec{k}'',\vec{k}} S^I_{M',M''} V^J_{\vec{k}',\vec{k}''} S^J_{M'',M} \frac{f_{\vec{k}''}}{-\epsilon_{\vec{k}} + \epsilon_{\vec{k}''}})
\end{aligned}$$

If, for a noncommutative model, (non spherically symetric potenials)

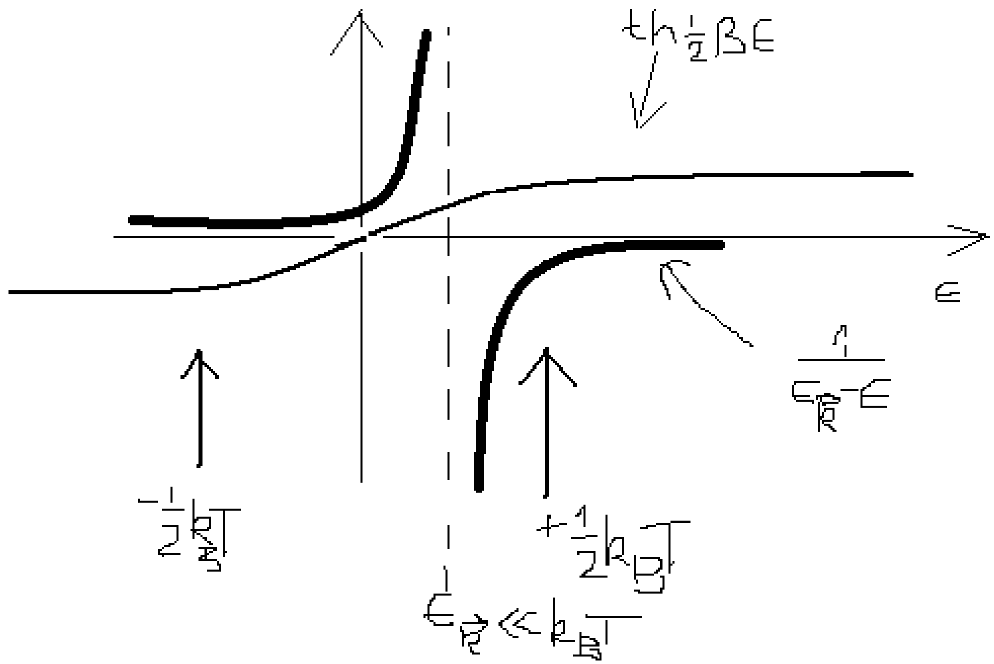
$$V^I_{\overrightarrow{k'},\overrightarrow{k}} = \sum_{L',L} V^I_{L',L}(k',k)Y_{L'}(\widehat{k'})Y_L(\widehat{k})$$

Take the simplest case L=0 and 1.Then the quantities $V^I_{L',L}$ are 2×2 matrecies and can be parametrize as

$$V^I_{L',L} = \sigma^I_{L',L}V^I$$

Then, for$\epsilon_{\overrightarrow{k}\,'} = \epsilon_{\overrightarrow{k},,} = \epsilon_F$ and

$$\sum_{\overrightarrow{k}\,''} \left(\frac{P}{\epsilon_{\overrightarrow{k}} - \epsilon_{\overrightarrow{k}\,''}}\right)\left(1 - 2f_{\overrightarrow{k}\,''}\right) = n(\epsilon_F)\int_{-D}^{D} d\epsilon \frac{P}{\epsilon_{\overrightarrow{k}} - \epsilon} th\left(\frac{1}{2}\beta\epsilon\right)$$



For $\epsilon_{\overrightarrow{k}} << k_B T$

$$\approx 2 \ln \frac{k_B T}{D}$$

For $\epsilon_{\overrightarrow{k}} >> k_B T$

$$\approx 2 \ln \frac{\epsilon_{\overrightarrow{k}}}{D}$$

Finally

$$\delta \widehat{T}^{(2)}_{\overrightarrow{k}'M';\overrightarrow{k},M}(\epsilon_{\overrightarrow{k}}) \approx \sum_{I,J,'M''} (V^I_{L',L''} V^J_{L'',L} - V^I_{L'',L} V^J_{L',L''}) \left(S^I_{M',M''} S^J_{M'',M} \right) \frac{k_F}{4\pi} \ln \left(\frac{D}{\epsilon_{\overrightarrow{k}}, k_B T} \right)$$

Evidently the perturbation series diverges. A way to summ up such logarithmically divergent series is to use scaling arguments as fallows

3.6 Poorman Scaling:

$$T(\epsilon; V_1 + \delta V_1, V_2 + \delta V_2, V_3 + \delta V_3; D + \delta D) = 0$$

In the case of magnetic impurities

$$\mathsf{T}_{\vec{k}',\vec{k}}(z) = [J_\perp(S^+\sigma^- + S^-\sigma^+) + J_\| S^z\sigma^z] - [J_\| J_\perp(S^+\sigma^- + S^-\sigma^+) + J_\|^2 S^z\sigma^z] n(\epsilon_F) \ln\frac{z}{D} + \ldots$$

$$\delta J \quad \perp \quad = -J_\perp J_\| n(\epsilon_f)\delta \ln\frac{z}{D} = -J_\perp J_\| N(\epsilon_F)\delta\ln\frac{\delta D}{D}$$

$$\delta J \quad \| \quad = -J_\|^2 N(\epsilon_F)\delta\ln\frac{z}{D} = -J_\|^2 N(\epsilon_F)\delta\ln\frac{\delta D}{D}$$

$$\frac{\partial J\perp}{\partial \ln D} = -nJ_\| J_\perp \quad , \quad \frac{\partial J_\|}{\partial \ln D} = -nJ_\|^2$$

For $J_\| = J_\perp = -J$ (J>0)

$$
\begin{aligned}
\frac{dJ}{dD} &= n\frac{J^2}{D} \\
\int_{J_0}^{J} dJ'\frac{1}{J'^2} &= n\int_{D_0}^{D}\frac{dD}{D} \\
-\frac{1}{J}+\frac{1}{J_0} &= n\ln\frac{D}{D_0} \\
J &= \frac{nJ_0}{1-nJ_0\ln\frac{D}{D_0}} \\
k_BT_K &= D_0e^{-\frac{1}{nJ_0}} \\
k_BT_K &= De^{-\frac{1}{nJ}}
\end{aligned}
$$

So K$_BT_K$ is a scale invariant low energy scale

As the temperature is lowered

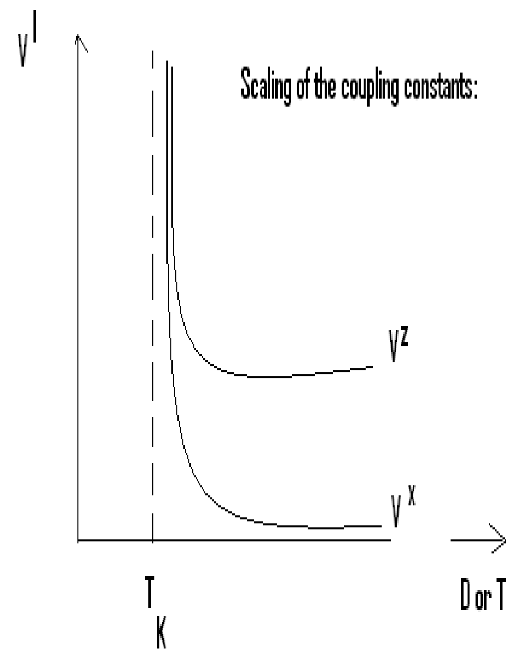
$$\textbf{nJ(T)}=\frac{1}{\ln\frac{T}{T_K}}$$

For the internal TLS, fallowing Zawadowski and Vladar , '92he we find that Scattering Aplitude scales to the strong couplig regime:

$$\begin{aligned}\frac{dV^x}{dx} &= -4V^yV^z\\ \frac{dV^y}{dx} &= -4V^zV^x\\ \frac{dV^z}{dx} &= -4V^xV^y\end{aligned}$$

where x=ln$\frac{D}{D_0}$

$$k_B T_K = D_0 \left(\frac{V_0^x}{4V_0^z}\right)^{\frac{1}{4V_0^z}}$$



V I
Scaling of the coupling constants:
V z
V x
T K
D or T

4 The chalange is to find a material specific, quantitative theory of the low energy scale

$$k_B T_K = (?) e^{-\frac{1}{n((\epsilon_F)J}}$$