

# Ab-initio theory of superconductivity: Density functional formalism, approximate functionals and first applications

M. Lüders<sup>1,2</sup>, M. A. L. Marques<sup>3,2</sup>, N. N. Lathiotakis<sup>3,2</sup>, A. Floris<sup>3,4</sup>, G. Profeta<sup>5</sup>,  
L. Fast<sup>6,2</sup>, A. Continenza<sup>5</sup>, S. Massidda<sup>4</sup>, E. K. U. Gross<sup>3,2</sup>

<sup>1</sup>*Daresbury Laboratory, Warrington WA4 4AD, United Kingdom*

<sup>2</sup>*Institut für Theoretische Physik, Universität Würzburg,  
Am Hubland, D-97074 Würzburg, Germany*

<sup>3</sup>*Institut für Theoretische Physik, Freie Universität Berlin,  
Arnimallee 14, D-14195 Berlin, Germany*

<sup>4</sup>*INFM SLACS, Sardinian Laboratory for Computational Materials Science and  
Dipartimento di Scienze Fisiche, Università degli Studi di Cagliari,  
S.P. Monserrato-Sestu km 0.700, I-09124 Monserrato (Cagliari), Italy*

<sup>5</sup>*C.A.S.T.I. - Istituto Nazionale Fisica della Materia (INFM) and  
Dipartimento di Fisica, Università degli studi dell'Aquila, I-67010 Coppito (L'Aquila) Italy*

<sup>6</sup>*SP Swedish National Testing and Research Institute, P.O.B. 857, S-501 15 Borås, Sweden*

## Abstract

A novel approach to the description of superconductors in thermal equilibrium is developed within a formally exact density-functional framework. The theory is formulated in terms of three “densities”: the ordinary electron density, the superconducting order parameter, and the diagonal of the nuclear  $N$ -body density matrix. The electron density and the order parameter are determined by Kohn-Sham equations that resemble the Bogoliubov-de Gennes equations. The nuclear density matrix follows from a Schrödinger equation with an effective  $N$ -body interaction. These equations are coupled to each other via exchange-correlation potentials which are universal functionals of the three densities. Approximations of these exchange-correlation functionals are derived using the diagrammatic techniques of many-body perturbation theory. The bare Coulomb repulsion between the electrons and the electron-phonon interaction enter this perturbative treatment on the same footing. In this way, a truly ab-initio description is achieved which does not contain any empirical parameters.

This new formalism is first applied to simple metals, to show the applicability to both weak and strong electron-phonon coupling. Further results for MgB<sub>2</sub>, and Al and Li under pressure will be presented.